NEST 2006

GALA
NEST ADVENTURE

The mathematics of surface motions can be challenging.

Mathematics for frontier technologies

The GALA team is pushing into an unexplored area of mathematics to develop general tools with a wide range of applications. If successful, this NEST Adventure project could lead to better understanding of human senses such as vision and hearing, new medical imaging techniques and new measuring instruments for materials science, spectroscopy and medical diagnosis.

Mathematics is vital to all physical and natural sciences as it provides methods and tools to help analyse and understand natural phenomena, and thus to model and predict their behaviour. The GALA project is exploring new instruments for mathematical geometric analysis that could be applied to modelling in a variety of scientific areas including human vision, robotics and materials science. This research covers 'Sub-Riemannian geometric analysis in Lie groups'.

Studying group behaviour

Group theory concerns systems and structures that obey common rules, such as geometrical systems. Lie groups are an important part of group theory that are used in mathematical analysis to describe the symmetry of structures in multi-dimensional space. Lie groups can be applied to non-Euclidean geometries, for example the geometrical relations on the surface of a sphere, known as elliptical geometry, are non-Euclidean. In multi-dimensional space, a Lie group describes a manifold; this is an abstract mathematical space in which simple localised properties can be studied to learn about a more complex global structure.

In Riemannian geometry, sub-Riemannian manifolds are particular manifolds that can describe constrained systems in classical mechanics, such as motions confined to a surface and the movement of robotic arms, where not all directions of movement are permitted; these are known as anisotropic systems. The aim of the GALA NEST project is to select real problems, for example in physics, bioengineering or financial engineering, that can be described as anisotropic systems and modelled using sub-Riemannian geometric analysis in Lie groups. The key to success lies in combining different expertise in areas of cutting-edge mathematical research to address this largely unexplored field.

Different disciplines, common goals

The success of the GALA project depends on achieving specific goals. The first is to understand the differential geometry of the relevant objects in Lie groups with a subriemannian geometry such as curves, surfaces and manifolds, and define their properties. The differential calculus used here is particularly applicable to the description of the curved surfaces found

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Lie groups are an important part of group theory since they can be applied to non-Euclidean geometries, such as the surface of a sphere. © Benoit Beauregard

AT A GLANCE

Official title
Geometrical analysis in Lie groups and applications

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on manifolds, and the partners from Bologna, Switzerland and Russia are experts in this field.

The second goal concerns 'geometric measure theory' which studies the stability of the objects on the manifolds; the two Swiss and two Finnish partners will contribute to this work. Once the basic objects in the Lie groups are understood, the next step is to develop the partial differential equations (PDEs) to describe their motion. It is the integration of differential geometry, geometric measure theory, and PDEs in Lie groups that will open up a wide range of applications for solving real problems.

The key to success lies in combining different expertise in areas of cutting-edge mathematical research.

Mind and matter applications

The generic modelling tools being developed in GALA are aimed at problems in emerging technologies. Recent advances in neurophysiology treat the brain as a highly anisotropic structure where cortical neurons are modelled as particles, like electrons and photons. GALA will model the electric charge flow from sensory neurons in the visual cortex and relate this flow to the geometric structures of perceptive representations – thus contributing to understanding of how vision occurs. Similarly, modelling the signal-processing functions of the cochlea – the sensory organ in human ears – will contribute to the mathematics of hearing and may lead to new hearing aids.

In materials science, impedance tomography is used to determine the bulk properties of a body through surface electrical measurements – a promising non-destructive technique with numerous potential industrial applications. Many material composites exhibit complex micro-geometries and the GALA instruments will help understand these – offering the prospect of characterising the macroscopic behaviour of materials using minimal information. If successful, the impact of GALA will be far-reaching, with applications ranging from financial engineering and stereo systems to quantum mechanics and spectroscopy.

SIXTH FRAMEWORK PROGRAMME